The Use of Knowledge in Social Algorithms

Rohit Parikh

City University of New York
Brooklyn College and CUNY Graduate Center

Beijing University
October 14, 2009

The peculiar character of the problem of a rational economic order is determined precisely by the fact that the knowledge of the circumstances of which we must make use never exists in concentrated or integrated form, but solely as the dispersed bits of incomplete and frequently contradictory knowledge which all the separate individuals possess.

The peculiar character of the problem of a rational economic order is determined precisely by the fact that the knowledge of the circumstances of which we must make use never exists in concentrated or integrated form, but solely as the dispersed bits of incomplete and frequently contradictory knowledge which all the separate individuals possess. The economic problem of society is thus not merely a problem of how to allocate "given" resources – if "given" is taken to mean given to a single mind which deliberately solves the problem set by these "data."

The peculiar character of the problem of a rational economic order is determined precisely by the fact that the knowledge of the circumstances of which we must make use never exists in concentrated or integrated form, but solely as the dispersed bits of incomplete and frequently contradictory knowledge which all the separate individuals possess. The economic problem of society is thus not merely a problem of how to allocate "given" resources – if "given" is taken to mean given to a single mind which deliberately solves the problem set by these "data." It is rather a problem of how to secure the best use of resources known to any of the members of society, for ends whose relative importance only these individuals know.

F. Hayek

Individualism and Economic Order

Plan of Talks

In the following we shall not present any new logics of knowledge. Many such logics already exist, and more are born every day. Rather the plan is to present situations where in real life and in popular media, reasoning about knowledge enters and to present sophisticated examples for you to try to formalize.

Making things explicit

Suppose you know p and $p \longrightarrow q$ then you ought to know q.
But you might not.

Making things explicit

Suppose you know p and $p \longrightarrow q$ then you ought to know q. But you might not.

But suppose I tell you q, then you should know q since you already knew p and $p \longrightarrow q$.

Mullah Nasruddin

Mullah Nasruddin is a Sufi character who always applies the right rules of inference, and the result is usually absurd.

Mullah Nasruddin

Mullah Nasruddin is a Sufi character who always applies the right rules of inference, and the result is usually absurd.

Sufism is a mystical branch of Islam, a little bit like Zen Buddhism

Mullah Nasruddin

Mullah Nasruddin is a Sufi character who always applies the right rules of inference, and the result is usually absurd.

Sufism is a mystical branch of Islam, a little bit like Zen Buddhism

Once, the Mullah was out for a walk and met some friends. In a generous mood, he invited them all for dinner.

Mullah Nasruddin

Mullah Nasruddin is a Sufi character who always applies the right rules of inference, and the result is usually absurd.

Sufism is a mystical branch of Islam, a little bit like Zen Buddhism

Once, the Mullah was out for a walk and met some friends. In a generous mood, he invited them all for dinner.

But then he said, "Let me go ahead and tell my wife that you are coming."

When he arrived at his house, his wife said, “Mullah, there is no food in the house and anyway, I cannot make dinner so fast!”

When he arrived at his house, his wife said, “Mullah, there is no food in the house and anyway, I cannot make dinner so fast!”

“I see,” said the Mullah. “Why don’t I hide upstairs and you tell my friends than I am not home.”

When he arrived at his house, his wife said, “Mullah, there is no food in the house and anyway, I cannot make dinner so fast!”

“I see,” said the Mullah. “Why don’t I hide upstairs and you tell my friends than I am not home.”

Pretty soon the friends arrived and the Mullah’s wife told them that the Mullah was not home.

“But we just saw the Mullah come in through the front door!”, said the friends.

“But we just saw the Mullah come in through the front door!”, said the friends.

“I am sorry, but the Mullah is not home”, said the wife.

“But we just saw the Mullah come in through the front door!”, said the friends.

“I am sorry, but the Mullah is not home”, said the wife.

“We don’t believe it, he must be home,” said the friends.

“But we just saw the Mullah come in through the front door!”, said the friends.

“I am sorry, but the Mullah is not home”, said the wife.

“We don’t believe it, he must be home,” said the friends.

The Mullah could not stand it any further and leaned out from the second floor window.

“But we just saw the Mullah come in through the front door!”, said the friends.

“I am sorry, but the Mullah is not home”, said the wife.

“We don’t believe it, he must be home,” said the friends.

The Mullah could not stand it any further and leaned out from the second floor window.

“Couldn’t I have gone out through the back door?”, said the Mullah.

Analysis

There are two ways of acquiring knowledge.

Analysis

There are two ways of acquiring knowledge.

We learn something from what someone tells us.

Analysis

There are two ways of acquiring knowledge.

We learn something from what someone tells us.

If someone tells us, “It is raining,” we learn that it is raining (if we trust that person).

Analysis

There are two ways of acquiring knowledge.

We learn something from what someone tells us.

If someone tells us, “It is raining,” we learn that it is raining (if we trust that person).

We learn from an event. If a drop of water falls on our head, we can also learn that it is raining.

Analysis

There are two ways of acquiring knowledge.

We learn something from what someone tells us.

If someone tells us, "It is raining," we learn that it is raining (if we trust that person).

We learn from an event. If a drop of water falls on our head, we can also learn that it is raining.

In the Mullah's case, he is arguing that he might not be home as he could have gone out through the back door.

Analysis

There are two ways of acquiring knowledge.

We learn something from what someone tells us.

If someone tells us, "It is raining," we learn that it is raining (if we trust that person).

We learn from an event. If a drop of water falls on our head, we can also learn that it is raining.

In the Mullah's case, he is arguing that he might not be home as he could have gone out through the back door.

But the event of his telling his friends that he could have gone out through the back door tells them that he is home.

Analysis

There are two ways of acquiring knowledge.

We learn something from what someone tells us.

If someone tells us, "It is raining," we learn that it is raining (if we trust that person).

We learn from an event. If a drop of water falls on our head, we can also learn that it is raining.

In the Mullah's case, he is arguing that he might not be home as he could have gone out through the back door.

But the event of his telling his friends that he could have gone out through the back door tells them that he is home.

Thus the Mullah story is a variant on what are called Moore sentences.

Analysis

There are two ways of acquiring knowledge.

We learn something from what someone tells us.

If someone tells us, “It is raining,” we learn that it is raining (if we trust that person).

We learn from an event. If a drop of water falls on our head, we can also learn that it is raining.

In the Mullah’s case, he is arguing that he might not be home as he could have gone out through the back door.

But the event of his telling his friends that he could have gone out through the back door tells them that he is home.

Thus the Mullah story is a variant on what are called Moore sentences.

If someone says, “It is raining but I do not believe that it is raining” then it is paradoxical.

I am indebted to Johan van Benthem for the following example

Three people A, B, C walk into a coffee shop. One of them orders cappuccino, one orders tea, and one orders icecream. The waiter goes away and after ten minutes *another* waiter arrives with three cups. “Who has the cappuccino?” “I do,” says A. “Who has the tea?” “I do,” says C.

Three people A, B, C walk into a coffee shop. One of them orders cappuccino, one orders tea, and one orders icecream. The waiter goes away and after ten minutes *another* waiter arrives with three cups. "Who has the cappuccino?" "I do," says A. "Who has the tea?" "I do," says C.

Will the waiter ask a third question?"

Consider the possible situations for waiter 2. They are

1) CTI, 2) CIT,
3) TCI, 4) TIC,
5) ICT, 6) ITC

Consider the possible situations for waiter 2. They are

1) CTI,   2) CIT,
3) TCI,   4) TIC,
5) ICT,   6) ITC

When A says that he has the cappuccino, 3,4,5,6 are eliminated.

Consider the possible situations for waiter 2. They are

1) CTI, 2) CIT,
3) TCI, 4) TIC,
5) ICT, 6) ITC

When A says that he has the cappuccino, 3,4,5,6 are eliminated. The waiter now has,

1) CTI, 2) CIT

Consider the possible situations for waiter 2. They are

1) CTI, 2) CIT,
3) TCI, 4) TIC,
5) ICT, 6) ITC

When A says that he has the cappuccino, 3,4,5,6 are eliminated. The waiter now has,

1) CTI, 2) CIT

When C says that he has the tea, 1 is eliminated.

Now 2 alone is left and the waiter knows that B has the icecream.

A butler enters a hotel room to clean it and make the bed, but he encounters a woman guest, coming out of the bathtub and not even wearing a towel.
“Excuse me, sir,” says the butler, and leaves the room.

A butler enters a hotel room to clean it and make the bed, but he encounters a woman guest, coming out of the bathtub and not even wearing a towel.
"Excuse me, sir," says the butler, and leaves the room.

Why did the butler say, "Excuse me, **sir**"?

A butler enters a hotel room to clean it and make the bed, but he encounters a woman guest, coming out of the bathtub and not even wearing a towel.
“Excuse me, sir,” says the butler, and leaves the room.

Why did the butler say, “Excuse me, **sir**”?

In the woman’s mind there were two possibilities.

A butler enters a hotel room to clean it and make the bed, but he encounters a woman guest, coming out of the bathtub and not even wearing a towel.
"Excuse me, sir," says the butler, and leaves the room.

Why did the butler say, "Excuse me, **sir**"?

In the woman's mind there were two possibilities.

S1 = "The butler saw her clearly"

A butler enters a hotel room to clean it and make the bed, but he encounters a woman guest, coming out of the bathtub and not even wearing a towel.
“Excuse me, sir,” says the butler, and leaves the room.

Why did the butler say, “Excuse me, **sir**”?

In the woman’s mind there were two possibilities.

S1 = “The butler saw her clearly”
S2 = “The butler did not see her clearly”

The butler’s remark eliminated S1 and saved her from embarrassment.

The dog who did not bark

In **The Silver Blaze** Sherlock Holmes investigates the case of a man killed on the moor, and a missing horse.

The dog who did not bark

In **The Silver Blaze** Sherlock Holmes investigates the case of a man killed on the moor, and a missing horse.

Sherlock Holmes arrives at the scene and carries out investigations.

The dog who did not bark

In **The Silver Blaze** Sherlock Holmes investigates the case of a man killed on the moor, and a missing horse.

Sherlock Holmes arrives at the scene and carries out investigations.

As he leaves, after finishing, his host asks for a hint of what happened.

The dog who did not bark

In **The Silver Blaze** Sherlock Holmes investigates the case of a man killed on the moor, and a missing horse.

Sherlock Holmes arrives at the scene and carries out investigations.

As he leaves, after finishing, his host asks for a hint of what happened.

"There was the curious incident of the dog in the night", says Holmes.

The dog who did not bark

In **The Silver Blaze** Sherlock Holmes investigates the case of a man killed on the moor, and a missing horse.

Sherlock Holmes arrives at the scene and carries out investigations.

As he leaves, after finishing, his host asks for a hint of what happened.

"There was the curious incident of the dog in the night", says Holmes.

"But the dog did nothing in the night", says the host.

The dog who did not bark

In **The Silver Blaze** Sherlock Holmes investigates the case of a man killed on the moor, and a missing horse.

Sherlock Holmes arrives at the scene and carries out investigations.

As he leaves, after finishing, his host asks for a hint of what happened.

"There was the curious incident of the dog in the night", says Holmes.

"But the dog did nothing in the night", says the host.

"That was the curious incident!", says Holmes.

The dog who did not bark

In **The Silver Blaze** Sherlock Holmes investigates the case of a man killed on the moor, and a missing horse.

Sherlock Holmes arrives at the scene and carries out investigations.

As he leaves, after finishing, his host asks for a hint of what happened.

"There was the curious incident of the dog in the night", says Holmes.

"But the dog did nothing in the night", says the host.

"That was the curious incident!", says Holmes.

Why did Holmes say what he did?

Some facts

1. The crime took place between t and t'.

Some facts

1. The crime took place between t and t'.
2. The horse was at location L at time t.

Some facts

1. The crime took place between t and t'.
2. The horse was at location L at time t.
3. The crime must have begun at L.

Some facts

1. The crime took place between t and t'.
2. The horse was at location L at time t.
3. The crime must have begun at L.
4. The perpetrator must have been at L between t and t'.

Some facts

1. The crime took place between t and t'.
2. The horse was at location L at time t.
3. The crime must have begun at L.
4. The perpetrator must have been at L between t and t'.
5. The dog was at location L between t and t'.

Some facts

1. The crime took place between t and t'.
2. The horse was at location L at time t.
3. The crime must have begun at L.
4. The perpetrator must have been at L between t and t'.
5. The dog was at location L between t and t'.
6. The dog would bark at any stranger during the night.
7. ______

Some facts

1. The crime took place between t and t'.
2. The horse was at location L at time t.
3. The crime must have begun at L.
4. The perpetrator must have been at L between t and t'.
5. The dog was at location L between t and t'.
6. The dog would bark at any stranger during the night.
7. ______
8. Conclusion: The perpetrator must be known to the dog.

Guilty Knowledge

The following story involves two characters, the C, or cheater and A, or accuser. The case was solved by detective (D) Columbo using reasoning about knowledge.

Guilty Knowledge

The following story involves two characters, the C, or cheater and A, or accuser. The case was solved by detective (D) Columbo using reasoning about knowledge.

A and C are business partners but C has been stealing from A and sending the money out of the country. A discovers the theft, accuses C and threatens to expose him.

Guilty Knowledge

The following story involves two characters, the C, or cheater and A, or accuser. The case was solved by detective (D) Columbo using reasoning about knowledge.

A and C are business partners but C has been stealing from A and sending the money out of the country. A discovers the theft, accuses C and threatens to expose him.

C decides to kill A but he needs to prepare. He makes a tape recording of C calling A's secretary S where C says to S, "Can I speak to A?"

Guilty Knowledge

The following story involves two characters, the C, or cheater and A, or accuser. The case was solved by detective (D) Columbo using reasoning about knowledge.

A and C are business partners but C has been stealing from A and sending the money out of the country. A discovers the theft, accuses C and threatens to expose him.

C decides to kill A but he needs to prepare. He makes a tape recording of C calling A's secretary S where C says to S, "Can I speak to A?"

C then goes to the gymnasium where A is working, and strangles him. But A is not weak, there is a struggle during which coffee is spilled and and scuff marks left on the polished gym floor.

Guilty Knowledge

The following story involves two characters, the C, or cheater and A, or accuser. The case was solved by detective (D) Columbo using reasoning about knowledge.

A and C are business partners but C has been stealing from A and sending the money out of the country. A discovers the theft, accuses C and threatens to expose him.

C decides to kill A but he needs to prepare. He makes a tape recording of C calling A's secretary S where C says to S, "Can I speak to A?"

C then goes to the gymnasium where A is working, and strangles him. But A is not weak, there is a struggle during which coffee is spilled and and scuff marks left on the polished gym floor.

C then dresses A in gym clothes, puts him on a weight lifting bench and puts heavy weights on his neck.

Guilty Knowledge

The following story involves two characters, the C, or cheater and A, or accuser. The case was solved by detective (D) Columbo using reasoning about knowledge.

A and C are business partners but C has been stealing from A and sending the money out of the country. A discovers the theft, accuses C and threatens to expose him.

C decides to kill A but he needs to prepare. He makes a tape recording of C calling A's secretary S where C says to S, "Can I speak to A?"

C then goes to the gymnasium where A is working, and strangles him. But A is not weak, there is a struggle during which coffee is spilled and and scuff marks left on the polished gym floor.

C then dresses A in gym clothes, puts him on a weight lifting bench and puts heavy weights on his neck.

C then leaves and goes to a party where he has invited S.

At the party C calls S from another phone and plays the recording of A's voice saying, "Can I speak to C?" S calls C to the phone.

At the party C calls S from another phone and plays the recording of A's voice saying, "Can I speak to C?" S calls C to the phone.

C then carries on a conversation with A (who is actually dead) and advises him not to go lift weights in the gym by himself.

At the party C calls S from another phone and plays the recording of A's voice saying, "Can I speak to C?" S calls C to the phone.

C then carries on a conversation with A (who is actually dead) and advises him not to go lift weights in the gym by himself.

The next day, A is found dead and the detective Columbo (D) is called.

At the party C calls S from another phone and plays the recording of A's voice saying, "Can I speak to C?" S calls C to the phone.

C then carries on a conversation with A (who is actually dead) and advises him not to go lift weights in the gym by himself.

The next day, A is found dead and the detective Columbo (D) is called.

D notices the scuff marks on the floor which make no sense as A was supposed to be by himself.

At the party C calls S from another phone and plays the recording of A's voice saying, "Can I speak to C?" S calls C to the phone.

C then carries on a conversation with A (who is actually dead) and advises him not to go lift weights in the gym by himself.

The next day, A is found dead and the detective Columbo (D) is called.

D notices the scuff marks on the floor which make no sense as A was supposed to be by himself.

Also, A's shoelaces are tied, not the way A would tie them himself, but the mirror image, as if someone else tied the shoelaces.

At the party C calls S from another phone and plays the recording of A's voice saying, "Can I speak to C?" S calls C to the phone.

C then carries on a conversation with A (who is actually dead) and advises him not to go lift weights in the gym by himself.

The next day, A is found dead and the detective Columbo (D) is called.

D notices the scuff marks on the floor which make no sense as A was supposed to be by himself.

Also, A's shoelaces are tied, not the way A would tie them himself, but the mirror image, as if someone else tied the shoelaces.

Conclusion, A was dressed in gym clothes **after** he was dead, and presumably by the killer.

The solution

Columbo then says to C, "The last that A was seen, he was wearing business clothes. When he was found he was wearing gym clothes."

The solution

Columbo then says to C, “The last that A was seen, he was wearing business clothes. When he was found he was wearing gym clothes.”

“Since we know that he was dressed in gym clothes after he was dead, how did **you** know that he was dressed in gym clothes when you “talked” to him on the phone?”

The solution

Columbo then says to C, “The last that A was seen, he was wearing business clothes. When he was found he was wearing gym clothes.”

“Since we know that he was dressed in gym clothes after he was dead, how did **you** know that he was dressed in gym clothes when you “talked” to him on the phone?”

“You are the killer!”

Numerical Foreheads

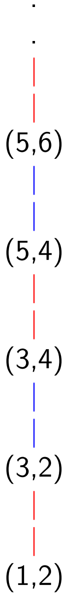
Two players Ann and Bob are told that the following will happen. Some positive integer n will be chosen and *one* of n, $n+1$ will be written on Ann's forehead, the other on Bob's. Each will be able to see the other's forehead, but not his/her own.

Note that each can see the other's number, but not their own. Thus if Ann has 5 and Bob has 6, then Ann knows that her number is either 5 or 7 and Bob knows that his number is either 6 or 4.

After this is done, they are asked repeatedly, beginning with Ann, if they know what their own number is.

Theorem 1: In those cases where Ann has the even number, the reponse at the nth stage will be, "my number is $n+1$", and in the other cases, the response at the $(n+1)$st stage will be "my number is $n+1$". In either case, it will be the person who *sees* the smaller number, who will respond first.

Start situation



Bob has just said, I don't know my number

.

.

(5,6)

(5,4)

(3,4)

(3,2)

~~(1,2)~~

Ann said no also

.

.

(5,6)

(5,4)

(3,4)

~~(3,2)~~

~~(1,2)~~

Bob said a second “no”

$\vdots$

(5,6)

(5,4)

$\cancel{(3,4)}$

$\cancel{(3,2)}$

$\cancel{(1,2)}$

Ann said a second “no”

⋮

(5,6)

~~(5,4)~~

~~(3,4)~~

~~(3,2)~~

~~(1,2)~~

Bob knows his number is 6

However, there is a serious defect in the argument in that both Ann and Bob's reasoning depends heavily on what the other one is thinking, including a consideration of what the other does not know. Ann's reasoning is justified if Bob thinks as she believes he does, and Bob's reasoning is justified if she thinks as he believes she does. But there is no guarantee that they do indeed think this way. How do we justify what each thinks and what each does and does not know?

We introduce the notion of interactive discovery system which gets around the issue of knowledge, and asks instead, What can they **do?**

We introduce the notion of interactive discovery system which gets around the issue of knowledge, and asks instead, What can they **do?**

Definition 4: An *IDS* (interactive discovery system) for M is a map

$f : W \times N^{+} \rightarrow \{\text{“}no\text{”}\} \cup W$ such that for each odd n, $f(s, n)$ (Ann's response at stage n) depends only on the $\equiv_1$ equivalence class of s and on $f(s, m)$ for $m < n$. For each even n, $f(s, n)$ depends only on the $\equiv_2$ equivalence class of s and on $f(s, m)$ for $m < n$.

Definition 5: The IDS f is *sound* if

- for all s, if $f(s, n) \neq$ "*no*", then $f(s, n) = s$.

Definition 5: The IDS f is *sound* if

- for all s, if $f(s,n) \neq$ "*no*", then $f(s,n) = s$.

The strategy that always says, "I don't know" is sound!

Definition 5: The IDS f is *sound* if

- for all s, if $f(s,n) \neq$ "*no*", then $f(s,n) = s$.

The strategy that always says, "I don't know" is sound!

We define $i_f(s) = \mu_n(f(s,n) \neq \text{"}no\text{"})$

and $p(s) = 1$ if $i_f(s)$ is odd and 2 if $i_f(s)$ is even.

Definition 5: The IDS f is *sound* if

- for all s, if $f(s,n) \neq$ "*no*", then $f(s,n) = s$.

The strategy that always says, "I don't know" is sound!

We define $i_f(s) = \mu_n(f(s,n) \neq \text{"}no\text{"})$

and $p(s) = 1$ if $i_f(s)$ is odd and 2 if $i_f(s)$ is even.

(Here μ stands for "least". $i_f(s) = \infty$ if $f(s,n)$ is always "no". We may drop the subscript f from i_f if it is clear from the context.)

Lemma 1: Let f be a sound IDS. Let $s \equiv_i t$, $i(s) = k < \infty$ and $p(s) = i$. Then $i(t) < k$ and $p(t) \neq i$.
Proof: At stage $i(s)$, i has evidence distinguishing between s and t. Since all previous utterances associated with s were "no", some previous utterance associated with t must have been nontrivial.

Lemma 1: Let f be a sound IDS. Let $s \equiv_i t$, $i(s) = k < \infty$ and $p(s) = i$. Then $i(t) < k$ and $p(t) \neq i$.
Proof: At stage $i(s)$, i has evidence distinguishing between s and t. Since all previous utterances associated with s were "no", some previous utterance associated with t must have been nontrivial.

Formally, $f(s, i(s)) = s \neq f(t, i(s))$. But $s \equiv_i t$. Hence $(\exists m < i(s))(f(s, m) \neq f(t, m))$. Since $m < i(s)$, $f(s, m) =$ "*no*" and so $f(t, m) \neq$ "*no*".

Thus $i(t) \leq m < i(s)$. Now, if $p(t) = i$, then, by a symmetric argument, we could prove also that $i(t) < i(s)$. But this is absurd. Hence $p(t) \neq i$. $\square$

Corollary: Suppose that $p(s) = i$ and there is a chain $s = s_1 \equiv_1 s_2 \equiv_2 s_3 \equiv_1 \ldots s_m$. Then $i(s) \geq m$.

Corollary: Suppose that $p(s) = i$ and there is a chain $s = s_1 \equiv_1 s_2 \equiv_2 s_3 \equiv_1 ...s_m$. Then $i(s) \geq m$.

Corollary: Suppose that there is a chain $s_1 \equiv_1 s_2 \equiv_2 s_3 \equiv_1 ...s_m \equiv_2 s_1$, with $m > 1$. Then $i(s_i) = \infty$ for all i.

Corollary: Suppose that $p(s) = i$ and there is a chain $s = s_1 \equiv_1 s_2 \equiv_2 s_3 \equiv_1 ... s_m$. Then $i(s) \geq m$.

Corollary: Suppose that there is a chain $s_1 \equiv_1 s_2 \equiv_2 s_3 \equiv_1 ... s_m \equiv_2 s_1$, with $m > 1$. Then $i(s_i) = \infty$ for all i.

Proof: If, say, $i(s_1) = k < \infty$, we would get $i(s_1) > i(s_2) > ... > i(s_m) > i(s_1)$, a contradiction. $\square$

Of course, the s_i are supposed to be distinct.

These strategies are *optimal*. E.g. we have

$$(6,5) \equiv_1 (4,5) \equiv_2 (4,3) \equiv_1 (2,3) \equiv_2 (2,1)$$

and hence $i(6,5)$ has a minimum value of 5, the value achieved by the strategy above.

A social structure with certain logical properties is a queue, like at a bus stop or in a bank.

- Someone who came earlier gets service earlier.
- Violations are easily detectable.

The problem of parking is a similar problem. A scarce resource needs to be allocated on the basis of some sort of priority, which, however, is difficult to determine.

When people are looking for parking in a busy area, they tend to cruise around until they find a space. There is no queue as such, but in general we do want that someone who arrives first should find a parking space and someone who arrives later may not.

When my students and I studied cruising for parking in a 15-block business district in Los Angeles, we found the average cruising time was 3.3 minutes, and the average cruising distance half a mile (about 2.5 times around the block). This may not sound like much, but with 470 parking meters in the district, and a turnover rate for curb parking of 17 cars per space per day, 8,000 cars park at the curb each weekday. Even a small amount of cruising time for each car adds up to a lot of traffic.

Over the course of a year, the search for curb parking in this 15-block district created about 950,000 excess vehicle miles of travel, equivalent to 38 trips around the earth, or four trips to the moon. And here's another inconvenient truth about underpriced curb parking: cruising those 950,000 miles wastes 47,000 gallons of gas and produces 730 tons of the greenhouse gas carbon dioxide. If all this happens in one small business district, imagine the cumulative effect of all cruising in the United States. Donald Shoup

Shoup regards this problem as one of incentive and suggests that parking fees be raised so that occupancy of street parking spaces is only 85%.

But perhaps this is really a knowledge problem?

Find a Place to Park on Your GPS – Spark Parking Makes it Possible

Navigation Developers Can Access Spark Parking Points of Interest Through New Tele Atlas ContentLink Program

San Francisco, CA, March 21, 2007

Running late for a meeting and worried about finding a place to park? Unhappy about paying outrageous valet parking fees at your favorite restaurant? These headaches will soon be a thing of the past. Spark Parking's detailed parking location information data is now available through the newly released Tele Atlas ContentLinkSM portal for application developers to incorporate into a range of GPS devices and location-based services and applications.

Spark Parking's detailed parking information provides the locations of every paid parking facility in each covered city – from the enormous multi-level garages to the tiny surface lots hidden in alleys. In addition, Spark Parking includes facility size, operating hours, parking rates, available validations, and many more details not previously available from any source. As a result, drivers will easily be able to find parking that meets their needs and budgets. http://www.pr.com/press-release/33381

Common Knowledge

Defined independently by Lewis and Schiffer. Used first in Game theory by Aumann.

Aumann showed that common knowledge implies same opinion.

Common Knowledge

Defined independently by Lewis and Schiffer. Used first in Game theory by Aumann.

Aumann showed that common knowledge implies same opinion.

Geanakoplos and Polemarchakis showed that communication between two agents leads to common knowledge and same opinion.

Common Knowledge

Defined independently by Lewis and Schiffer. Used first in Game theory by Aumann.

Aumann showed that common knowledge implies same opinion.

Geanakoplos and Polemarchakis showed that communication between two agents leads to common knowledge and same opinion.

Parikh and Krasucki showed that among n agents communicating in pairs, common opinion about some quantity can come about without most agents communicating with others.

Aumann's argument

Column

	$v_{1,1}$	$v_{1,2}$	$v_{1,3}$	$v_{1,4}$
	$v_{2,1}$	$v_{2,2}$	$v_{2,3}$	$v_{2,4}$
Row	$v_{3,1}$	$v_{3,2}$	$v_{3,3}$	$v_{3,4}$
	$v_{4,1}$	$v_{4,2}$	$v_{4,3}$	$v_{4,4}$

Now Row's value v is

$v_1 = (1/4)[v_{1,1,} + v_{1,2} + v + 1, 3 + v_{1,4}]$

And Column's value w is

$w_1 = (1/4)[(v_{1,1,} + v_{2,1} + v_{3,1} + v_{4,1}]$

Now Row's value v is
$v_1 = (1/4)[v_{1,1,} + v_{1,2} + v + 1, 3 + v_{1,4}]$
And Column's value w is
$w_1 = (1/4)[(v_{1,1,} + v_{2,1} + v_{3,1} + v_{4,1}]$

Since these values are common knowledge,
$v_1 = v_2$ where $v_2 = (1/4)[v_{2,1,} + v_{2,2} + v + 2, 3 + v_{2,4}]$
and similarly for v_3 and v_4.

Now Row's value v is
$v_1 = (1/4)[v_{1,1,} + v_{1,2} + v + 1, 3 + v_{1,4}]$
And Column's value w is
$w_1 = (1/4)[(v_{1,1,} + v_{2,1} + v_{3,1} + v_{4,1}]$

Since these values are common knowledge,
$v_1 = v_2$ where $v_2 = (1/4)[v_{2,1,} + v_{2,2} + v + 2, 3 + v_{2,4}]$
and similarly for v_3 and v_4.

Thus v_1 must equal
$(1/16)[\Sigma v_{i,j} : i \leq 4, j \leq 4]$
and similarly for w_1.

Now Row's value v is
$v_1 = (1/4)[v_{1,1,} + v_{1,2} + v + 1, 3 + v_{1,4}]$
And Column's value w is
$w_1 = (1/4)[(v_{1,1,} + v_{2,1} + v_{3,1} + v_{4,1}]$

Since these values are common knowledge,
$v_1 = v_2$ where $v_2 = (1/4)[v_{2,1,} + v_{2,2} + v + 2, 3 + v_{2,4}]$
and similarly for v_3 and v_4.

Thus v_1 must equal
$(1/16)[\Sigma v_{i,j} : i \leq 4, j \leq 4]$
and similarly for w_1.
Thus $v = w$.

Using Aumann's reasoning,
Milgrom and Stokey proved a famous
No Trade theorem!

If A is selling a stock to B, and B is buying it, then obviously A thinks the stock will go down and B thinks it will go up. But this fact is common knowledge! By a proof based on Aumann, it cannot be common knowledge that they have different views of the stock and the sale cannot take place.

“I’d never join any club that would have me for a member”

Groucho Marx

But what if the value is not common knowledge?

Will communication help?

GP argument

Column

Row				
	2	3	5	4
	7	8	9	10
	3	2	5	4
	5	4	3	2

At this point Row announces that her expected value is 3.5, and column eliminates row 2

Column

	2	3	5	4
Row	~~7~~	~~8~~	~~9~~	~~10~~
	3	2	5	4
	5	4	3	2

Now column announces that his value is 3.33, and row eliminates columns 2,3

Column

	2	~~3~~	~~5~~	4
Row	~~7~~	~~8~~	~~9~~	~~10~~
	3	~~2~~	~~5~~	4
	5	~~4~~	~~3~~	2

Now Row announces his value as 3 = (2+4)/2 and Column eliminates row 3, 4, announcing his value as 2.

Column

	2	3	5	4
	7	8	9	10
Row	3	2	5	4
	5	4	3	2

At this point Row eliminates column 4, also announces his value at 2, and they have consensus.

Column

Row				
	2	3	5	4
	7	8	9	10
	3	2	5	4
	5	4	3	2

A brief overview of the [PK] result:
Suppose we have n agents connected in a strongly connected graph. They all share initial probability distribution, but have now received, each of them, a finite amount of private information. Thus their estimate of the probability of some event or the expected value of some random variable v may now be different.

Let g be a function which, at stage n picks out a sender $s(n)$ and a recipient $r(n)$. $s(n)$ sends his latest value of v to $r(n)$ who then revises *her* valuation of v.

If the graph G is strongly connected, and for each pair of connected agents i, j, i repeatedly sends his value of v to j, then eventually all estimates of the value of v become equal.

Parikh-Krasucki result

$$
\begin{tikzcd}
\mathbf{4} \arrow[d] & \mathbf{3} \arrow[l] \\
\mathbf{1} \arrow[r] & \mathbf{2} \arrow[u]
\end{tikzcd}
$$

Sketch of proof of the P-K result

Each agent i has a personal partition $\mathcal{P}_i$. Let $\mathcal{P}$ be the common refinement of the $\mathcal{P}_i$. The $\mathcal{P}_i$ are finite and so is $\mathcal{P}$.

Sketch of proof of the P-K result

Each agent i has a personal partition $\mathcal{P}_i$. Let $\mathcal{P}$ be the common refinement of the $\mathcal{P}_i$. The $\mathcal{P}_i$ are finite and so is $\mathcal{P}$. Each time an agent receives information, (s)he deletes at least one piece of $\mathcal{P}$ from her current set of possibilitities.

Sketch of proof of the P-K result

Each agent i has a personal partition $\mathcal{P}_i$. Let $\mathcal{P}$ be the common refinement of the $\mathcal{P}_i$. The $\mathcal{P}_i$ are finite and so is $\mathcal{P}$. Each time an agent receives information, (s)he deletes at least one piece of $\mathcal{P}$ from her current set of possibilitities.

After a finite number of steps each agent has removed as many pieces as she is ever going to and the situation stabilizes.

Sketch of proof of the P-K result

Each agent i has a personal partition $\mathcal{P}_i$. Let $\mathcal{P}$ be the common refinement of the $\mathcal{P}_i$. The $\mathcal{P}_i$ are finite and so is $\mathcal{P}$. Each time an agent receives information, (s)he deletes at least one piece of $\mathcal{P}$ from her current set of possibilitities.

After a finite number of steps each agent has removed as many pieces as she is ever going to and the situation stabilizes.

The non-trivial part of the proof consists of showing that at that stage, all agents have the same value of the parameter in question.

History Based Knowledge

On Monday Jack writes to Ann that he got a dog (**D**) E_1

History Based Knowledge

On Monday Jack writes to Ann that he got a dog (**D**) E_1
On Wednesday Ann receives his letter, E_2

History Based Knowledge

On Monday Jack writes to Ann that he got a dog (**D**) E_1
On Wednesday Ann receives his letter, E_2
On Thursday, Jack looks at the calendar and sees that three days have passed since he wrote, E_3

History Based Knowledge

On Monday Jack writes to Ann that he got a dog (**D**) E_1
On Wednesday Ann receives his letter, E_2
On Thursday, Jack looks at the calendar and sees that three days have passed since he wrote, E_3

$$E_1 \longrightarrow E_2 \longrightarrow E_3$$

History Based Knowledge

On Monday Jack writes to Ann that he got a dog (**D**) E_1
On Wednesday Ann receives his letter, E_2
On Thursday, Jack looks at the calendar and sees that three days have passed since he wrote, E_3

$$E_1 \longrightarrow E_2 \longrightarrow E_3$$

Suppose that a letter takes at most three days to arrive. Then on Wednesday, Ann knows D, but Jack does not know that Ann knows D.
On Thursday, Jack knows that Ann knows that D.

See no Evil, Hear no Evil

A pretty woman (Eve) shoots a man dead.

See no Evil, Hear no Evil

A pretty woman (Eve) shoots a man dead.

Wally, who is blind, hears a shot.

See no Evil, Hear no Evil

A pretty woman (Eve) shoots a man dead.

Wally, who is blind, hears a shot.

Dave, who is deaf, sees a woman leave in a hurry (his back was turned when she fired)

See no Evil, Hear no Evil

A pretty woman (Eve) shoots a man dead.

Wally, who is blind, hears a shot.

Dave, who is deaf, sees a woman leave in a hurry (his back was turned when she fired)

Together they know who committed the murder. But neither of them knows it by himself.

A global history is the sequence of all events which happen.
The corresponding local history for an agent i, is all the events (or aspects of them) which i 'sees'.
The protocol is the set of all possible global histories.
Suppose an agent sees local history h, and X is the set of all global histories which are compatible with h.
If some property P is true of all histories in X, then the agent knows P.

Example 1: Uma is a physician whose neighbour is ill. Uma does not know and has not been informed. Uma has no obligation (as yet) to treat the neighbour.

Example 1: Uma is a physician whose neighbour is ill. Uma does not know and has not been informed. Uma has no obligation (as yet) to treat the neighbour.

Example 2: Uma is a physician whose neighbour Sam is ill. The neighbour's daughter Ann comes to Uma's house and tells her. Now Uma does have an obligation to treat Sam, or perhaps call in an ambulance or a specialist.

Example 1: Uma is a physician whose neighbour is ill. Uma does not know and has not been informed. Uma has no obligation (as yet) to treat the neighbour.

Example 2: Uma is a physician whose neighbour Sam is ill. The neighbour's daughter Ann comes to Uma's house and tells her. Now Uma does have an obligation to treat Sam, or perhaps call in an ambulance or a specialist.

The global history contained the event E of Sam being sick, but until Uma was told, she did not know it and did not know that she needed to act.

The Model

In the model developed by Pacuit, Parikh and Cogan, the history of the world is a line in a tree of all possible histories. At each moment of time, the agents are at some branch point, and some agent has the move. This agent can choose one among the various trees beginning at the branch point, if the agent knows that one subtree has better outcomes than the other subtrees at that branch point, then the agent has the obligation to choose that subtree.

The subtree where Uma treats the sick man is better than the subtree where she does not. But until she is informed she does not know this. Once she is informed, she acquires an obligation.

The Kitty Genovese Murder

"Along a serene, tree-lined street in the Kew Gardens section of Queens, New York City, Catherine Genovese began the last walk of her life in the early morning hours of March 13, 1964.....As she locked her car door, she took notice of a figure in the darkness walking towards her. She became immediately concerned as soon as the stranger began to follow her.

'As she got of the car she saw me and ran,' the man told the court later, 'I ran after her and I had a knife in my hand.... I could run much faster than she could, and I jumped on her back and stabbed her several times,' the man later told the cops."

Many neighbours saw what was happening, but no one called the police.

“Mr. Koshkin wanted to call the police but Mrs. Koshkin thought otherwise. ‘I didn’t let him,’ she later said to the press, ‘I told him there must have been 30 calls already.’ ”

“When the cops finished polling the immediate neighbourhood, they discovered at least 38 people who had heard or observed some part of the fatal assault on Kitty Genovese.”[1]

Some 35 minutes passed between Kitty Genovese being attacked and someone calling the police, why?

---

[1] This quote is from the article ‘A cry in the night: the Kitty Genovese murder’, by a police detective, Mark Gado, and appears on the web in *Court TV’s Crime Library.*

Gricean Implicature

Gricean Implicature

A: My car is out of gasoline.

Gricean Implicature

A: My car is out of gasoline.

B: There is a gas station around the corner

Gricean Implicature

A: My car is out of gasoline.

B: There is a gas station around the corner

The assumption is that B is co-operating with A and would not say what he said unless he knew that the gas station was (likely to be) open.

But, can we always believe what others tell us?

But, can we always believe what others tell us?

Sally is applying to Rayco for a job and Rayco asks if her ability is high or low.

		Rayco	
		High	**Low**
Sally	**High**	(3,3)	(0,0)
	Low	(0,0)	(2,2)

.

Sally has nothing to gain by lying about her qualifications and Rayco can trust her.

		Rayco	
		High	**Low**
Sally	**High**	(3,3)	(0,0)
	Low	(3,0)	(2,2)

.

Sally has nothing to **lose** by lying about her qualifications and Rayco cannot trust her.

The extent to which one agent (the listener) can believe
another agent (the speaker)
depends on how much they have in common.

Obama and Rev. Jeremiah Wright

During the presidential campaign of 2008, the issue of Jeremiah Wright came up. Obama had been going to Wright's church for many years.

Obama and Rev. Jeremiah Wright

During the presidential campaign of 2008, the issue of Jeremiah Wright came up. Obama had been going to Wright's church for many years.

But Wright had made some anti-white remarks and also uttered the famous phrase, "God damn America!"

Obama and Rev. Jeremiah Wright

During the presidential campaign of 2008, the issue of Jeremiah Wright came up. Obama had been going to Wright's church for many years.

But Wright had made some anti-white remarks and also uttered the famous phrase, "God damn America!"

Obama's association with Wright was a political problem for Obama, and finally Obama dissociated himself from Wright.

Something interesting has happened recently in the kerfuffle between Barack Obama and his putative pastor, Jeremiah Wright.

Obama denounced comments made by Wright at the NAACP and at the Press Club.

Something interesting has happened recently in the kerfuffle between Barack Obama and his putative pastor, Jeremiah Wright.

Obama denounced comments made by Wright at the NAACP and at the Press Club.

Wright responded, "It went down very simply. He's a politician. I'm a pastor. We speak to two different audiences. And he says what he has to say as a politician. I say what I have to say as a pastor. Those are two different worlds. I do what I do, he does what politicians do. So what happened in Philadelphia where he had to respond to the sound bites, he responded as a politician."

So, it is important when people speak, to keep in mind that they often have an axe to grind

Thank You!